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Principles of Transpose in the Fixed Point Theory for Cone Metric Spaces

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Abstract. This paper presents new **principles of transpose** in the fixed point theory as for example: Let X be a nonempty set and let $\mathfrak{C}$ be an arbitrary formula which contains terms $x, y \in X$, $\leq$, $+$, $\preccurlyeq$, $\oplus$, $T : X \to X$, and ρ. Then, as assertion of the form: For every T and for every $\rho(x, y) \in \mathbb{R}^0_+ := [0, +\infty)$ the following fact

(A) $\quad \mathfrak{C}\big(x, y \in X, \leq,\ +,\ T,\ \rho\big)$ implies T has a fixed point

is a theorem if and only if the assertion of the form: For every T and for every $\rho(x, y) \in C$, where C is a cone of the set G of all cones, the following fact in the form

(TA) $\quad \mathfrak{C}\big(x, y \in X, \preccurlyeq,\ \oplus,\ T,\ \rho\big)$ implies T has a fixed point

is a theorem. Applications of the principles of transpose in nonlinear functional analysis and fixed point theory are numerous.

1. Introduction and history

The concept of an abstract metric space, introduced by M. Fréchet in 1905, furnishes the common idealization of a large number of mathematical, physical and other scientific constructs in which the notion of a *distance* appears.

The objects under consideration may be most varied. The may be points, functions, sets, and even the subjective experiences of sensations. A generalization which was first introduced by K. Menger in 1942 and, following him, is called a *statistical metric space*.

In 1934 Đ. Kurepa defined *pseudodistancional spaces*, with the nonnumerical distance, which play an important role in nonlinear numerical analysis (see: L. Collatz [2]). After that several authors investigated the distance functions taking values in partially ordered sets (A. Appert,

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M. Fréchet, J. Colmez, R. Doss, Ky Fan, and others in the year's 40's and 50's).

Concept of transversal spaces with the nonnumerical transverse were introduced in 1998 by Tasković as a nature extension of Fréchet's, Kurepa's, and Menger's spaces in well-know sense. The transversal spaces play an important role in nonlinear functional analysis as and in numerical analysis.

An example of pseudodistancial (as and transversal) spaces is so-called *cone of a metric space* (or *cone metric space*). For the cone metric space we formulate principles of transpose.

Let $E := (E, +)$ be a topological vector space. A subset P of E is called a **cone** iff P is a closed, nonempty and $P \neq \{0\}$; if $a, b \in \mathbb{R}$ $(a, b \geq 0)$ and $x, y \in P$ then $ax + by \in P$; and $P \cap (-P) = \{0\}$.

For a given cone $P \subset E$, we define a *partial ordering* $\preccurlyeq$ with respect to P by $x \preccurlyeq y$ if and only if $y - x \in P$. We shall write $x \prec y$ if $x \preccurlyeq y$ and $x \neq y$; also, $x \boxed{\preccurlyeq} y$ means that $y - x \in int\ P$, where $int\, P$ denotes the interior of P.

The cone P is called **normal** if there is a number $\sigma > 0$ such that for all $x, y \in E$ we have $\|x\| \leq \sigma\|y\|$ whenever $\theta \preccurlyeq x \preccurlyeq y$.

Let X be a nonempty set. In this sense, suppose that the mapping $\rho : X \times X \to P \subset E$ has all the metric axioms (i.e., $\rho[x, y] = \theta := 0$ if and only if $x = y$, $\rho[x, y] = \rho[y, x]$, and $\rho[x, y] \preccurlyeq \rho[x, z] \oplus \rho[z, y]$ as in the papers: Kurepa [7], Collatz [2], or Sikorski [9].

In the same manner, ρ is called a *cone metric* on X, and $X := (X, \rho, \oplus)$ is called **cone metric space**, where $\oplus = +$ in the topological vector space E. Thus ρ satisfies all the axioms of transvsersal spaces with the nonnumerical transverse (as and all axioms of Kurepa's pseudodistantial spaces, see: Kurepa [7]).

2. PRINCIPLES OF TRANSPOSE

We are now in a position to formulate our main theorems as principles of transpose for cone metric spaces and further.

Theorem 1 (Principle of Transpose). *Let X be a nonempty set and let $\mathfrak{C}$ be an arbitrary formula which cintains terms $x, y \in X$, $\leq$, $+$, $\preccurlyeq$, $\oplus$, $T : X \to X$ and ρ. Then, an assertion of the form: For every T and for every $\rho(x, y) \in \mathbb{R}^0_+$ the following fact in the form*

(A) $\qquad \mathfrak{C}\big(x, y \in X, \leq,\ +,\ T,\ \rho\big)$ *implies T has a fixed point*

is a theorem if and only if the assertion of the form: For every T and for every $\rho(x, y) \in C$, where C is a cone of the set G of all cones, the following fact in the form

(TA) $\qquad \mathfrak{C}\big(x, y \in X, \preccurlyeq,\ \oplus,\ T,\ \rho\big)$ *implies T has a fixed point* *is a theorem.* (We recall that a statement is *Local Principle of Transpose* if (A) and (TA) hold only via term of the form $x \in X$).

In concetion wirh the preceding statement if facts (A) and (TA) to substitute with the following facts in the forms as

$$\text{(A}'\text{)} \qquad \mathfrak{C}\big(x, y \in X, \leq,\ +,\ T,\ \rho\big) \quad \text{implies} \quad \mathfrak{M}(T),$$

and

$$\text{(TA}'\text{)} \qquad \mathfrak{C}\big(x, y \in X, \preccurlyeq,\ \oplus,\ T,\ \rho\big) \quad \text{implies} \quad \mathfrak{M}(T),$$

respectively, where the property $\mathfrak{M}(T)$ denotes all conclusions of the **Banach contraction principle**:

1) *T has a unique fixed point $\xi \in X$,*
2) *$x_n = T^n(x) \to \xi$ for every $x \in X$,* and
3) *there exists an estimate of the rapidity of convergence*;

then, Theorem 1 also to remain holds.

If the mapping $T : X \to X$ has the properties 1), 2) and 3), then we say that T is a **Banach's mapping**.

Proof of Theorem 1. Suppose that (A) holds and suppose that (TA) not hold. Then there exist T and $\rho \in C$ for every cone $C \in G$ such that $\mathfrak{C}(x, y \in X, \preccurlyeq, \oplus, T, \rho)$ holds and that T not have fixed point. But, for the case $C = \mathbb{R}^0_+$ the fact (A) holds, contradicting to the preceding fact. Thus (TA) holds.

Conversely of the preceding conditions, applying Axiom of Choice to this situation we obtain that (A) holds, i.e., (TA) implies (A). The proof is complete. □

Taking one consideration with another, as an immediate fact from the preceding statement, we have directly the following result.

Theorem 2 (Cone Principle of Transpose)**.** *Let X be a nonempty set and let $\mathfrak{C}$ be an arbitrary formula which contains terms $x, y \in X$, $\leq$, $+$, $\preccurlyeq$, $\oplus$, $f_i : X \to X$ $(i = 1, \ldots, k)$ for a fixed number $k \in \mathbb{N}$, and ρ. Then, an assertion of the form: For every f_i $(i = 1, \ldots k)$ and for every $\rho(x, y) \in \mathbb{R}^0_+$ the following fact in the form*

$$\text{(E)} \qquad \begin{array}{l} \mathfrak{C}\big(x, y \in X, \leq,\ +,\ f_i\ (i = 1, \ldots, k), \rho\big) \\ \textit{implies } f_i\ (i = 1, \ldots, k) \textit{ have a coincidence point} \end{array}$$

is a theorem if and only if the assertion of the form: For every f_i $(i = 1, \ldots, k)$ and for every $\rho(x, y) \in C$, where C is a cone of the set G of all cones, the following fact in the form

$$\text{(R)} \qquad \begin{array}{l} \mathfrak{C}\big(x, y \in X, \preccurlyeq,\ \oplus,\ f_i\ (i = 1, \ldots, k), \rho\big) \\ \textit{implies } f_i\ (i = 1, \ldots, k) \textit{ have a coincidence point} \end{array}$$

is a theorem. (The local form of this statement we obtain whenever (E) and (R) hold only via term of the form $x \in X$).

We notice that in the preceding statement if facts (E) and (R) to substitute with the following facts in the forms as

(E′) $\quad \mathfrak{C}\big(x, y \in X, \leq,\ +,\ f_i\ (i = 1, \ldots, k), \rho\big) \quad \text{implies} \quad \mathfrak{M}(f_i),$

and

(R′) $\quad \mathfrak{C}\big(x, y \in X, \preccurlyeq,\ \oplus,\ f_i\ (i = 1, \ldots, k), \rho\big) \quad \text{implies} \quad \mathfrak{M}(f_i),$

respectively, where the property $\mathfrak{M}(f_i)$ is a form of:

1) f_i $(i = 1, \ldots, k)$ have a common fixed point,
2) there exists a countable sequence which converges to a common fixed point of f_i $(i = 1, \ldots, k)$, and
3) f_i $(i = 1, \ldots, k)$ have a unique common fixed point.

3. CONSEQUENCES OF THE PRINCIPLES OF TRANSPOSE

Let $X := (X, \rho)$ be a cone metric space and let $\{x_n\}_{n\in\mathbb{N}}$ be a sequence in X. Let $x \in X$, if for every $c \in E$ with $\theta \boxed{\preccurlyeq} c$ there is $n_0 \in \mathbb{N}$ such that $\rho[x_n, x] \boxed{\preccurlyeq} c$ for every $n \geq n_0$, then $\{x_n\}_{n\in\mathbb{N}}$ is said to be **convergent** and it converges to x, i.e., x is limit of $\{x_n\}_{n\in\mathbb{N}}$. If for any $c \in E$ with $\theta \boxed{\preccurlyeq} c$, there is $n_0 \in \mathbb{N}$ such that, $\rho[x_n, x_m] \boxed{\preccurlyeq} c$ for all $n, m \geq n_0$, then $\{x_n\}_{n\in\mathbb{N}}$ is called a **Cauchy sequence** in X. If every Cauchy sequence is convergent in X, then X is called a **complete cone metric space**.

There exist several applications of the preceding principles of transpose. In the fixed point theory, theorems of the forms (A) and (E) are usually proved first. However, theorems of the forms (TA) and (R) are more general (in the sense of sufficiency), so the proofs of the theorems are usually similar. Using our principles of transpose, we are able to state at once the theorems (A) and (TA), i.e., (E) and (R) depending which of the theorems is wanted. We shall illustrate the preceding principles of transpose with the several examples.

Theorem 3 (Banach [1]). *Let $X := (X, \rho)$ be a complete metric space with the metric ρ and $T : X \to X$ such that there exists $\lambda \in [0, 1)$ satisfying*

(B) $$\rho[T(x), T(y)] \leq \lambda \rho[x, y] \quad \textit{for all} \quad x, y \in X,$$

then the mapping T has a unique fixed point in X.

This statement is well-known as a part of Banach contraction principle in 1922. Also, this statement is a form of the fact (A) in Theorem 1. Applying Principle of Transpose *directly* we obtain the following statement as a fact of the form (TA).

Theorem 3a. *Let $X := (X, \rho)$ be a complete cone metric space. Suppose that the mapping $T : X \to X$ satisfies the following condition*

$$\rho[T(x), T(y)] \preccurlyeq \lambda\rho[x, y] \quad \text{for all} \quad x, y \in X,$$

where $\lambda \in [0, 1)$ is a constant. Then the mapping T has a unique fixed point in X.

Theorem 4 (Goebel [5], Jungck [6])**.** *Let $X := (X, \rho)$ be a complete metric space and suppose that the mappings $f, g : X \to X$ satisfy*

$$\rho[f(x), f(y)] \le \lambda\rho[g(x), g(y)] \quad \text{for all} \quad x, y \in X,$$

where $\lambda \in [0, 1)$ is a constant. If the range of g contains the range of f and $g(X)$ is a closed subspace of X, then f and g have a unique point of coincidence in X.

This statement as a coincidence theorem which Goebel proved in 1968 has recently received some attention. In 1976 Jungck generalized Goebel's coincidence theorem using a pair of commuting mappings. We can to write down that this statement is a form of the fact (E) in Theorem 2. Applying Theorem 2 (Cone Principle of Transpose) directly we obtain the following statement as a fact of the form (R).

Theorem 4a. *Let $X := (X, \rho)$ be a cone metric space. Suppose that the mappings $f, g : X \to X$ satisfy*

$$\rho[f(x), f(y)] \preccurlyeq \lambda\rho[g(x), g(y)] \quad \text{for all} \quad x, y \in X,$$

where $\lambda \in [0, 1)$ is a constant. If the range of g contains the range f and $g(X)$ is a complete subspace of X, then f and g have a unique point of coincidence in X.

Suppose that f and g are two self-mappings of a metric (or a cone metric) space X. The pair $\{f, g\}$ is **asymptotically regular** at $x_0 \in X$ iff $\{\rho[x_n, x_{n+1}]\}_{n\in\mathbb{N}}$ is convergent to θ, where in the sequel: $x_1 = f(x_0)$, $x_2 = g(x_1), \ldots, x_{2n+1} = f(x_{2n})$, $x_{2n+2} = g(x_{2n+1}), \ldots$

Theorem 5 (Tasković [15])**.** *Let f, g be two self-mappings of a complete metric space $X := (X, \rho)$ such that for each $x, y \in X$ the following inequality holds*

$$\rho[f(x), g(y)] \le \varphi\big(\text{diameter}\{x, y, f(x), g(y)\}\big),$$

where the existing a nondecreasing mapping $\varphi : \mathbb{R}^0_+ \to \mathbb{R}^0_+ = [0, +\infty)$ has the following property

$$(\varphi) \qquad \limsup_{z\to t+0} \varphi(z) < t \quad \text{for every } t > 0,$$

and suppose that the pair $\{f, g\}$ is asymptotically regular at $x_0 \in X$, then the following sequence in the form as

$$(1) \qquad x_1 = f(x_0), x_2 = g(x_1), \ldots, x_{2n+1} = f(x_{2n}), x_{2n+2} = g(x_{2n+1}), \ldots$$

converges to $\xi \in X$ *such that* $f(\xi) = g(\xi) = \xi$. *If* $f(q) = q$ *and* $g(p) = p$, *then* $p = q = \zeta$.

This statement as a coincidence theorem which Tasković proved in 1978 has recently received some attention. If for the class (φ) of functions $\varphi : \mathbb{R}_+^0 \to \mathbb{R}_+^0$ we defined a subclass of the form that $\varphi : P := \text{Cone}(X) \to \text{Cone}(X)$ is continuous (or weakly continuous) and increasing, then we can to write down that this statement is a form of the fact (E) in Theorem 2. Applying Theorem 2 (Cone Principle of Transpose) directly we obtain the following statement as a fact of the form (R).

Theorem 5a. *Let* f, g *be two self-mappings of a complete cone metric space* $X := (X, \rho)$ *and let* $P(X)$ *be a normal cone such that there exists a continuous increasing mapping* $\varphi : P \to P$ *with the property* $\varphi(t) \prec t$ *for every* $t \neq \theta$ *such that*

$$\rho[f(x), g(y)] \preccurlyeq \varphi\big(\textit{diameter}\{x, y, f(x), g(y)\}\big),$$

for all $x, y \in X$. *If the pair* $\{f, g\}$ *is asymptotically regular at* $x_0 \in X$, *then* f *and* g *have a unique point of coincidence* $\xi \in X$ *such that* $f(\xi) = g(\xi) = \xi$.

Let $X := (X, \rho)$ be a metric space and $T : X \to X$, where $\rho : X \times X \to \mathbb{R}_+^0 := [0, +\infty)$. In 1985 we investigated the concept of TCS-convergence in a space X, i.e., a metric space X satisfies the *condition of TCS-cenvergence* iff $x \in X$ and if $\rho[T^n(x), T^{n+1}(x)] \to 0$ (as $n \to \infty$) implies that $\{T^n(x)\}_{n\in\mathbb{N}}$ has a convergent subsequence.

Theorem 6 (Tasković [13]). *Let* T *be a mapping of a metric space* $X := (X, \rho)$ *into itself, where* X *satisfies the condition of TCS-convergence. Suppose that for all* $x, y \in X$ *there exist a sequence of functions* $\{\alpha_n(x, y)\}_{n\in\mathbb{N}}$ *such that* $\alpha_n(x, y) \to 0$ $(n \to \infty)$ *and a positive integer* $m(x, y)$ *such that*

$$\rho[T^n(x), T^n(y)] \leq \alpha_n(x, y) \quad \textit{for all} \quad n \geq m(x, y),$$

where $x \mapsto \rho[x, T(x)]$ *is a lower semicontinuous function, then* T *has a unique fixed point* $\xi \in X$ *and* $T^n(x) \to \xi$ *as* $n \to \infty$ *for each* $x \in X$.

In the next, a cone metric space $X := (X, \rho)$ satisfies the *condition of TCS-convergence* iff $x \in X$ and if $\rho[T^n(x), T^{n+1}(x)] \to \theta$ (as $n \to \infty$) implies that $\{T^n(x)\}_{n\in\mathbb{N}}$ has a convergent subsequence.

We notice, the preceding statement is a form of the fact (A) in Theorem 1. Applying Theorem 1 (Principle of Transpose) *directly* we obtain the following statement as a fact of the form (TA).

Theorem 6a. *Let* T *be a mapping of a cone metric space* $X := (X, \rho)$ *into itself, where* X *satisfies the condition of TCS-convergence. Suppose that for all* $x, y \in X$ *there exist a sequence of functions* $\{\alpha_n(x, y)\}_{n\in\mathbb{N}}$ *such that* $\alpha_n(x, y) \to \theta$ $(n \to \infty)$ *and a positive integer* $m(x, y)$ *such that*

$$\rho[T^n(x), T^n(y)] \preccurlyeq \alpha_n(x, y) \quad \textit{for all} \quad n \geq m(x, y),$$

where $x \mapsto \rho[x, T(x)]$ *is a lower semicontinuous function, then* T *has a unique fixed point* $\xi \in X$ *and* $T^n(x) \to \xi$ *as* $n \to \infty$ *for each* $x \in X$.

A characterization of the class of cone contractive mappings. In recent years a great number of papers has appeared presenting a various generalization of the well known Banach-Picard contraction principle (via linear and nonlinear conditions).

The far-reaching consequence practical way of Banach's contraction principle in 1922 lies in as for the fact that the underlying space is quite complete metric, while the conclusion is strong including even error estimates.

It has a long history in nonlinear functional analysis and in the fact concept of "asymptotic contraction mappings" is suggested by one of the earliest concepts of Banach's principle attributed to Italian mathematician R. Caccioppoli, which in 1930 observed that a mapping $T : X \to X$ on a complete metric space (X, ρ) has a unique fixed point if for each $n \geqslant 1$ there exists a constant $c_n > 0$ for all $x, y \in X$ such that

$$\rho\big[T^n(x), T^n(y)\big] \le c_n \rho[x, y] \quad \text{and} \quad \sum_{n=1}^{\infty} c_n < +\infty.$$

In Tasković [14] we introduce the concept of **contraction mapping** T of a metric space X, i.e., of a mapping $T : X \to X$ such that for all $x, y \in X$ there exists a sequence of nonnegative real functions $\mathfrak{A}_{n,r}(x, y)$ with $\mathfrak{A}_{n,r}(x, y) \to 0$ $(r \geqslant n \to \infty)$ and a positive integer $m(x, y)$ such that

$$\text{(Ta)} \qquad \rho[T^n(x), T^r(y)] \leqslant \mathfrak{A}_{n,r}(x, y) \quad \text{for all} \quad r > n \geqslant m(x, y).$$

Also, in Tasković [14] we introduced the concept of **σ-contraction** T of a metric space X into itself, i.e., of a mapping $T : X \to X$ such that for all $x, y \in X$ there exist numbers $C_n(x, y) > 0$ and $K(x, y) > 0$ such that

$$\rho[T^n(x), T^n(y)] \le C_n(x, y)K(x, y), \quad \text{for all} \quad n \in \mathbb{N},$$

where $\sum_{n=1}^{\infty} C_n(x, y)$ is a convergent series for all $x, y \in X$.

Theorem 7 (Tasković [14, p. 48]). *Necessary and sufficient conditions that a selfmap* T *of a metric space* $X := (X, \rho)$ *has the following properties:*

(a) T *has a unique fixed point* $\zeta \in X$,

(b) $x_n = T^n(x) \to \zeta$ $(n \to \infty)$ *for every* $x \in X$, *and*

$$\rho[T^n x, \zeta] \leqslant \mathfrak{A}_n(x, Tx), \quad \textit{for} \quad n \geqslant m(x),$$

where $x \mapsto \mathfrak{A}_n(x, Tx)$ *are real nonnegative functions with* $\mathfrak{A}_n(x, Tx) \to 0$ $(n \to \infty)$ *are the following ones:*

(e) X *is* T*-orbital complete,*

(f) T *is orbital continuous, and*

(g) T *is contraction mapping, where*

$$\mathfrak{A}_{n,r}(x, y) = \max\big\{\mathfrak{A}_n(x, Tx), \mathfrak{A}_r(y, Ty)\big\}.$$

The main fact for further applications of this result is an investigation of solvability of an integral equation of the form

$$\text{(IE)} \qquad u(x) = v(x) + \lambda \int_{\alpha}^{x} K(x,t,u(t))\,dt \quad \text{for} \quad \alpha \le x \le \beta,$$

by successive approximation, where $\lambda \in \mathbb{R}$ is an arbitrary parameter, $v(x)$ is a given continuous function in the compact interval $[\alpha,\beta]$ and $K(m,s,x)$ is continuous for $m,s \in [\alpha,\beta]$. See: Tasković [14] and Tişe [17].

In this sense, a cone metric space $X := (X,\rho)$ is said to be **orbital complete** (or T-*orbital complete*) iff every Cauchy sequence which is contained in $\mathcal{O}(x) := \{x, Tx, T^2(x), \ldots\}$ for some $x \in X$ converges in X.

A mapping T of a cone metric space X into itself is said to be **orbital continuous** if T for every $x \in X$ is a continuous mapping via orbits, i.e., if $T(T^{n(k)}x) \to T\zeta$ whenever $T^{n(k)}x \to \zeta \in X$ for every $x \in X$ and $k \to \infty$.

From the preceding facts and applying Theorem 1 (Principle of Transpose) we obtain directly the following characterization of a class of transversal contractive mappings on cone metric space.

In this part we introduce the concept of **cone contraction mapping** T of a cone metric space X, i.e., of a mapping $T: X \to X$ such that for all $x,y \in X$ there exists a sequence of functions $\mathfrak{A}_{n,r}(x,y)$ in the cone $C(X)$ with $\mathfrak{A}_{n,r}(x,y) \to \theta$ $(r \geqslant n \to \infty)$ and a positive integer $m(x,y)$ such that

$$\text{(Ta)} \qquad \rho[T^n(x), T^r(y)] \preccurlyeq \mathfrak{A}_{n,r}(x,y) \quad \text{for all} \quad r > n \geqslant m(x,y).$$

Theorem 7a. *Necessary and sufficient conditions that a selfmap T of a cone metric space $X := (X,\rho)$ has the following properties:*

(a) *T has a unique fixed point $\zeta \in X$,*

(b) *$x_n = T^n(x) \to \zeta$ $(n \to \infty)$ for every $x \in X$, and*

$$\rho[T^n x, \zeta] \preccurlyeq \mathfrak{A}_n(x, Tx), \quad \textit{for} \quad n \geqslant m(x),$$

where $x \mapsto \mathfrak{A}_n(x,Tx)$ are functions with $\mathfrak{A}_n(x,Tx) \to \theta$ $(n \to \infty)$, are the following ones:

(e) *X is T-orbital complete,*

(f) *T is orbital continuous, and*

(g) *T is cone contraction mapping, where*

$$\mathfrak{A}_{n,r}(x,y) = \sup\big\{\mathfrak{A}_n(x,Tx), \mathfrak{A}_r(y,Ty)\big\}.$$

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